parameter is a pure imaginary, and there are no losses in either waveguide. Thus $k=jc$, $\gamma_{1,2}=j\beta_{1,2}$. Upon inserting these conditions and solving for the magnitude of the field in the auxiliary waveguide we find that

$$|E_2| = \frac{2C}{\sqrt{(\beta_1-\beta_2)^2+4C^2}} \cdot \sin\left(\frac{\sqrt{(\beta_1-\beta_2)^2+4C^2}}{2}x\right). \quad (15)$$

As the slab is moved across the main waveguide, both β_1 and C are affected. The phase constant is determined by means of (5) and the change in coupling is calculated in the following manner. With the slab at $d=0$, the two phase constants are equal and the coupling is found from $|E_2| = \sin C_0X$. For each subsequent position of the slab, C_0 is multiplied by the factor H_{zs}/H_{z0} given in (12).

REFERENCES

[1] A. M. Woodward, "Transmission in waveguides," *Wireless Engr.*, vol. 24, pp. 192–196; July, 1947.
[2] C. G. Montgomery, R. H. Dicke, and E. M. Purcell, "Principles of Microwave Circuits," M.I.T. Rad. Lab. Ser., McGraw-Hill Book Co., Inc., New York, N. Y., vol. 8, pp. 385–389; 1948.
[3] P. H. Vartanian, W. P. Ayres, and A. L. Helgesson, "Propagation in dielectric slab loaded rectangular waveguide," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-6, pp. 215–222; April, 1958.
[4] A. D. Berk, "Variational principles for electromagnetic resonators and waveguides," IRE TRANS. ON ANTENNAS AND PROPAGATION, vol. AP-4, pp. 104–111; April, 1956.
[5] I. Tetsuro, and M. Yasuto, "Universal solution of rectangular waveguide loaded with a dielectric slab," *J. Inst. Elec. Commun. Engrs., Japan*, vol. 43, pp. 943–947; September, 1960.
[6] S. E. Miller, "Coupled wave theory and waveguide applications," *Bell Sys. Tech. J.*, vol. 33, pp. 661–719; May, 1954.
[7] G. E. Schafer and R. W. Beatty, "A method for measuring the directivity of directional couplers," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-6, pp. 419–422; October, 1958.
[8] A. G. Fox, S. E. Miller, and M. T. Weiss, "Behavior and applications of ferrites in the microwave region," *Bell Sys. Tech. J.*, vol. 34, pp. 5–103; January, 1955.

Millimeter Wavelength Resonant Structures*

R. W. ZIMMERER†, MEMBER, IRE, M. V. ANDERSON†, G. L. STRINE†, AND Y. BEERS†, MEMBER, IRE

***Summary*—This paper discusses the construction of millimeter wave Fabry-Perot resonators, using both planar and spherical reflectors. It also discusses the equivalent circuits of planar reflectors and the method of obtaining efficient power transfer into the resonators.**

THIS PAPER is a further report on the work on millimeter wave Fabry-Perot interferometers that was started in this laboratory by Culshaw.[1-5] These interferometers have become of wide interest because of their use as resonators in optical and millimeter wave masers. These resonators have many other potential uses as spectrometers, refractometers, and wave meters.

In the millimeter region the ratio of wavelength to the mirror dimensions, although small compared to unity, is much larger than in the optical region. Therefore, diffraction losses in the millimeter region tend to be much larger. At the same time modes are separated more widely, and it is usually possible to work with a single mode. In the optical region mirrors are made of semisilvered surfaces or by multilayered dielectric surfaces. As is well known,[6] with such mirrors large reflectivity is incompatible with low resonance transmission loss. Culshaw realized that in the millimeter region other techniques allowing the achievement of both objectives were practical. He evolved a scheme of drilling an array of holes in metallic sheets and started work with metal films with photoetched holes deposited on dielectric slabs. We have further developed this technique and have used thin perforated metal foils stretched on frames. This technique appears to be the best available for use with plane reflectors.

For many applications Fox and Li[7] and Boyd and Gordon[8] have demonstrated the superiority of con-

* Received October 23, 1962; revised manuscript received November 30, 1962. A condensed version of this paper was presented at the PGMTT National Symposium in Boulder, Colo., on May 22, 1962.

† Radio Standards Laboratory, National Bureau of Standards, Boulder, Colo.

[1] W. Culshaw, "Reflectors for a microwave Fabry-Perot interferometer," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-7, pp. 221–228; April, 1959.

[2] W. Culshaw, "High resolution millimeter wave Fabry-Perot interferometer," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-8, pp. 182–189; March, 1960.

[3] W. Culshaw, "Resonators for millimeter and submillimeter wavelengths," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-9, pp. 135–144; March, 1961.

[4] W. Culshaw, "Millimeter wave techniques," *Advances in Electronics and Electron Phys.*, vol. 15, pp. 197–263; 1961.

[5] W. Culshaw, "Measurement of permittivity and dielectric loss with a millimetre-wave Fabry-Perot Interferometer," *Proc. IEE*, vol. 109, pt. B, Suppl. No. 23, pp. 820–826; 1961.

[6] M. Born and E. Wolf, "Principles of Optics," Pergamon Press, Inc., New York, N. Y., Sec. 7.6; 1959.

[7] A. G. Fox and T. Li, "Resonant modes in a maser interferometer," *Bell Sys. Tech. J.*, vol. 40, pp. 453–488; March, 1961.

[8] G. D. Boyd and J. P. Gordon, "Confocal multimode resonator for millimeter through optical wavelength masers," *Bell Sys. Tech. J.*, vol. 40, pp. 489–508; March, 1961.

cave mirrors over planar ones because of the greatly reduced diffraction losses and the greater ease of alignment. In some of the instruments to be described later we have applied their ideas to the millimeter region. Marcuse,[9] in independently applying them to the millimeter region, coupled the resonator to a waveguide by a single hole. In principle, diffraction losses should be greater with this type of coupling, and probing the periphery of the field with absorbing objects tends to confirm this hypothesis. With both types of coupling we have obtained Q values within a factor of two or three of that computed for solid metal reflectors with no diffraction losses. It is probable that the slightly greater diffraction losses of the single hole feed are compensated by increased reflector losses with the multihole coupling. Unless a high uniformity of field is required for some special application, the single hole feed is to be preferred because of its simplicity.

The Properties of Thin Perforated Reflectors

With particular reference to the Stark spectrometer to be described below, we were concerned with the problem of obtaining good power transfer into the resonator while preserving high field uniformity and high reflectivity. In attacking this problem we adopted the policy of using simple approximate theory as a guide to the extent that such theory is valid and then of relying upon experiment to compensate for higher-order effects. One such theory is the impedance theory.

To check the validity of the impedance theory, we set up the transmission experiment shown in the upper part of Fig. 1. The perforated plate is placed between

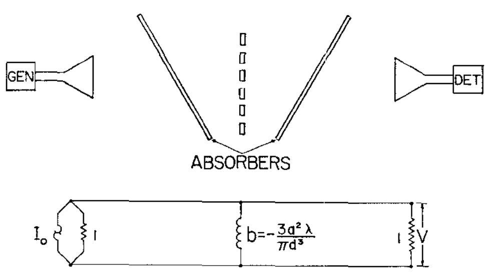

Fig. 1—Transmission through a lossless perforated thin plate.

coaxial transmitting and receiving horns. The signal received with the plate in the beam is compared to that with it absent. Standing waves between the plate and either horn are greatly reduced by placing absorbers at oblique angles on either side of the plate. A magazine is a convenient absorber, and the attenuation can be varied by choosing the number of pages. The significance of the experimental transmission coefficients was enhanced by separately varying the horn spacing and the plate position along the axis. These variations produced no more than 0.5 db change in the transmission coefficients. Transmission factor was taken to be the change in power at the detector as measured by a bolometer.

In the analysis of the experiment, it is assumed that the situation is described by a wave incident upon the plate, a wave reflected into the transmitter space, and a wave transmitted into the detector space. Furthermore, the simplifying assumption is made that these are all plane (TEM) waves of a single plane polarization. Since the beam is bounded in cross section, other modes must actually be present, but the experimental data show that they are of small amplitude. Such assumptions also imply that the plate does not depolarize the beam. In principle, if the rows of holes make oblique angles with the field vectors, depolarization can take place. We have observed no evidence of such depolarization nor any dependence of transmission factor upon orientation. Nevertheless, all measurements reported here were made with the rows of holes aligned with the field vectors.

If the space on the side of the plate away from the source is unbounded, if losses are negligible, and if only one mode is propagated at distances sufficiently removed from the plate, the plate can be represented by a susceptance in a transmission line analogy. Under these conditions, the power carried by the reflected wave on the side facing the source and the power carried by the transmitted wave on the other side must equal the power carried by the incident wave. In a transmission line such a condition is produced by connecting a susceptance across a line of infinite length. A mathematical proof seems hardly necessary to establish this equivalence, but such proofs can be found in textbooks.[10] However, a mathematical treatment is required to evaluate the susceptance. Such a one[11–13] shows that, if the holes are of diameter d and in a rectangular array with spacings a and c, the normalized susceptance is approximately

$$b = -\frac{3ac\lambda}{\pi d^3}. \tag{1}$$

Eq. (1) is an approximation valid when a, c, and d are small compared to λ. Higher-order terms have been derived by Munushian,[13] who shows that (1) is the appropriate expression for an array of holes. It is interesting to note that when either a or c exceeds λ, the

[9] D. Marcuse, "Maser oscillation observed from HCN maser at 88.6 kMc," Proc. IRE (*Correspondence*), vol. 49, pp. 1706–1707; November, 1961.

[10] J. C. Slater, "Microwave Electronics," D. Van Nostrand Co., Inc., New York, N. Y., Sec. 6.5; 1950.
E. Ginzton, "Microwave Measurements," McGraw-Hill Book Co., Inc., New York, N. Y., Sec. 6.4; 1957.

[11] C. G. Montgomery, R. H. Dicke, and E. M. Purcell, "Principles of Microwave Circuits," McGraw-Hill Book Co., Inc., New York, N. Y., Sec. 6.11; 1948.

[12] N. Marcuwitz, "Waveguide Handbook," McGraw-Hill Book Co., Inc., New York, N. Y., ch. 5; 1951.

[13] J. Munushian, "Electromagnetic Propagation Characteristics of Space Arrays of Apertures-in-Metal Discontinuities and Complementary Structures," Electronics Research Lab., University of California at Berkeley, Lab. Rept., Ser. No. 60, Issue 126; September, 1954.

theory breaks down qualitatively as well, because the amplitudes produced by various holes interfere constructively to give maxima of radiation off the axis. Under these conditions, the plate acts as a grating producing more than one order of constructive interference.

In cases which are of interest in this work b is large in magnitude compared to unity. Then it can be easily shown that the transmission coefficient is

$$T = -20 \log_{10} \frac{2\pi d^3}{3ac\lambda} \quad \text{expressed in decibels.} \tag{2}$$

The argument of the logarithm is twice the reciprocal of the susceptance. If the theory is valid, all experimental points should lie on a single straight line when the transmission coefficient is plotted against the logarithm of twice the susceptance. With a single plate, the susceptance is varied by varying λ. Such data are plotted along with the theoretical line in Fig. 2. With two plates, the points lie on lines of the same slope as the theoretical line but slightly below it. The latter fact can probably be attributed to losses in the plates, which are neglected in the theory. The curve representing the third plate crosses the theoretical line at high values of the argument (small values of λ). However, this plate had comparatively large spacings between the holes, and at short wavelengths the assumptions of the theory are not fulfilled. With all three plates, the holes were in a square array, and thus $c=a$. The data can be considered to be in reasonable agreement with the impedance theory. The use of this approximate theory as a guide in the design of interferometers is therefore justified.

The experimental data displayed in Fig. 2 were all obtained on thin metal foils stretched on frames: that is, foils whose thickness was small compared to a free-space wavelength. Brief qualitative consideration of transmission line theory indicates that the presence of lossless dielectric backing causes a reduced admittance mismatch, resulting in more energy being transmitted through the perforated plate than would be predicted from (2). When the frequency is such as to cause the thickness of the dielectric to be an integral number of dielectric half wavelengths, the transmission coefficient should agree with (2).

If a plane wave is normally incident upon a plane metal surface and if Γ is the reflection coefficient at the generator, the fraction of the incident power which is dissipated in the metal is

$$t_0 = 1 - |\Gamma|^2.$$

If a slab of lossless dielectric of thickness l and dielectric constant K is placed in contact with the metal on the generator side, the fraction of the power dissipated in the metal is increased unless, of course, the thickness of the dielectric happens to be exactly an integral number of half wavelengths.

In general, the fraction of power dissipated is

$$t = \frac{K}{1 + (K-1)\cos^2\theta} t_0, \tag{2a}$$

where

$$\theta = \frac{2\pi l}{\lambda_1}.$$

t_0 is the value of t in the absence of dielectric, and λ_1 is the wavelength in the dielectric. Fig. 3 shows the relative power dissipated in the metal for three frequently used dielectrics as a function of θ. The following values of dielectric constants have been used: 1) plate glass 9.5, 2) quartz 3.78, and 3) rexolite 2.2.

The power transmitted through a perforated sheet is therefore increased by the factor t/t_0 given by (2a). The preceding discussion applies strictly to solid metal plates. However, our experience indicates that it holds as an excellent approximation for perforated plates.

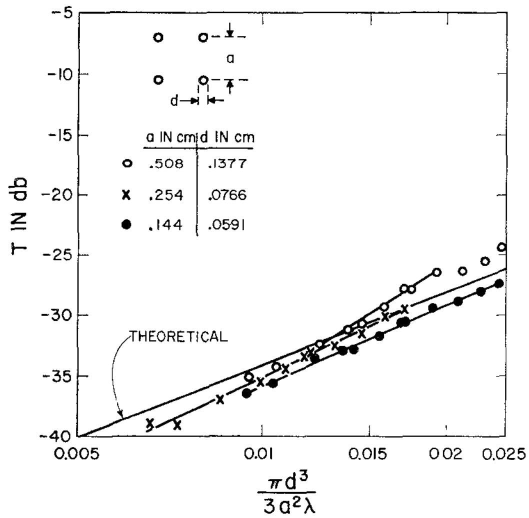

Fig. 2—Transmission measurements of three perforated thin plates.

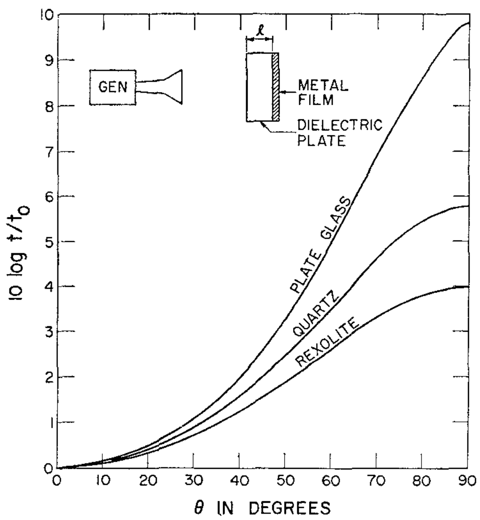

Fig. 3—Increase in absorption in metal film when deposited on a dielectric.

When the thickness of the metal plate is appreciable, it is represented by a three-terminal network, as is well known.[12] If the rows of holes are oblique to the field vectors such that depolarization effects must be considered, it can be supposed that a plate can be represented by a network having two pairs of input terminals and two pairs of output terminals.

The Resonant Frequencies of Parallel Plate Interferometers

In all practical applications of the Fabry-Perot interferometer, observation is made along the central axis peripendicular to the plates. What is considered as a resonance from the microwave point of view corresponds to a maximum in intensity at the center of the field of view in an optical interference pattern. In elementary texts, the behavior is explained in terms of the multiple reflection of plane waves between the reflectors. It is assumed that these plane waves have the same wavelength λ_0 and phase velocity as in an unbounded medium. If the plates are perfectly reflecting, the condition for resonance is then

$$\lambda_0 = (2D/m) \tag{3}$$

where D is the separation and m is any positive integer. The frequency is given by dividing λ_0 into the phase velocity.

Such a description is inadequate to explain all of the recent observations. Schawlow and Townes,[14] in proposing the use of the Fabry-Perot interferometer as a resonator for optical masers, suggested that it should be considered as a rectangular box with four open sides. A transmission line with both ends short circuited resonates at the same frequencies as when both ends are open circuited except that the positions of the nodes and antinodes are interchanged. Analogously, if the interferometer has rectangular plates, each of dimensions A and B separated by a distance D, it can be expected to resonate at the same frequencies as a closed rectangular box of the same dimensions where the free-space wavelengths are given very accurately by

$$\lambda_{m,n,p} = 2\left[\frac{m^2}{D^2} + \frac{n^2}{A^2} + \frac{p^2}{B^2}\right]^{-1/2}, \tag{4}$$

where m, n, and p are nonzero integers.

The theoretical work of Fox and Li[7] investigated the field patterns and showed that they do indeed differ from plane waves. Symmetry considerations require that n and p be odd integers for modes which are observed with coupling which is symmetrical about the central axis of the instrument. In the following we assume this symmetry.

In most practical situations, the second and third terms in the brackets of (4) are small compared to the first. In cases where they may be completely neglected, it can be seen that $\lambda_{m,n,p}$ becomes equal to λ_0 obtained in the elementary theory. In conventional optical situations these terms are generally so small that modes of the same m and differing n and p lie so close together as not to be resolved, and the elementary theory is adequate. However, with masers, the resolution is such that the frequencies emitted as the result of simultaneous oscillation in several of these modes can be resolved. The modes with the lowest values of n and p, namely unity, have the highest Q and give rise to the strongest maser lines. Also they generally produce the strongest and sharpest resonances in the interferometers described in the paper. The original report on the helium-neon gas maser[15] contains excellent experimental verification of the validity of (4).

In that paper the strong signals at 150-Mc intervals are due to beating of modes with different m's but all with $n=1$ and $p=1$. The weaker peaks displaced by 1.5 Mc are due to beats between a mode described by $n=1$ and $p=1$ and a mode with a different m and either n or p equal to 3 while the other of these two quantities remains equal to 1. Quantitatively these values are consistent with (4) and the geometry of the apparatus, in which $B=A$.

For many purposes it is convenient to employ an approximation for (4) by expressing D in terms of λ_0 by (3) and retaining only first-order terms in a binominal expression. B is set equal to A, since this condition usually prevails. Then

$$\lambda_{m,n,p} = \lambda_0\left[1 - \frac{(n^2 + p^2)\lambda_0^2}{8A^2}\right]. \tag{5}$$

Since n and p are never zero, the second term on the right is not identically zero and it must be considered if the Fabry-Perot resonator is used as a wavemeter of the highest available accuracy. λ_0 may be determined by measuring the displacement of one plate between major resonances. Then a correction can be determined by substituting this value into the second term of (5). A can be determined from geometry or by measuring the frequency shift between a main resonance for which $n=1$ and $p=1$ to a subsidiary one where one or both has a higher value.

The Q of Parallel Plate Resonators

The unloaded Q of a parallel plate resonator is given approximately by the following well-known expression[1]

$$Q = \frac{m\pi}{1 - |\Gamma|^2} \tag{6}$$

wherein Γ is the amplitude reflection coefficient of the surfaces. In (6) diffraction losses are neglected, and in the numerator an approximation has been made by

[14] A. L. Schawlow and C. H. Townes, "Infrared and optical masers," *Phys. Rev.*, vol. 112, pp. 1940–1949; December, 1958.

[15] A. Javan, W. R. Bennett, Jr., and R. Herriott, "Population inversion and continuous optical maser oscillation in a gas discharge containing a He-Ne mixture," *Phys. Rev. Lett.*, vol. 6, pp. 106–110; February, 1961.

setting $|\Gamma|^2 \sim 1$ in a more exact expression. This latter approximation is valid under all circumstances which are of interest. This equation has been derived on the basis of the elementary plane wave theory.

According to (6), Q should increase linearly with m (or D). However, at larger spacings diffraction becomes important and limits the Q obtainable. This situation is illustrated by the experimental data presented in Fig. 4. The data lie on a straight line until D becomes comparable to A, which is equal to 14 cm in this instrument. Then Q falls below the line. In this experiment the holes were such as to give weak coupling. The measured Q is therefore essentially the unloaded Q.

The straight line of greater slope in Fig. 4 represents the calculated value of Q if the plates were made of solid aluminum. The fact that the slope of the experimental line is a factor of 8 or so less indicates that the surface losses are correspondingly greater than for solid plates. This is not unreasonable because the plates used in this experiment had thin films barely one skin depth thick deposited on glass. The dielectric slab also increases the natural loss of the metal film as given by (2a). Data obtained in this laboratory on thicker plates indicate that surface losses are no more than 2 or 3 times greater than calculated for solid plates.

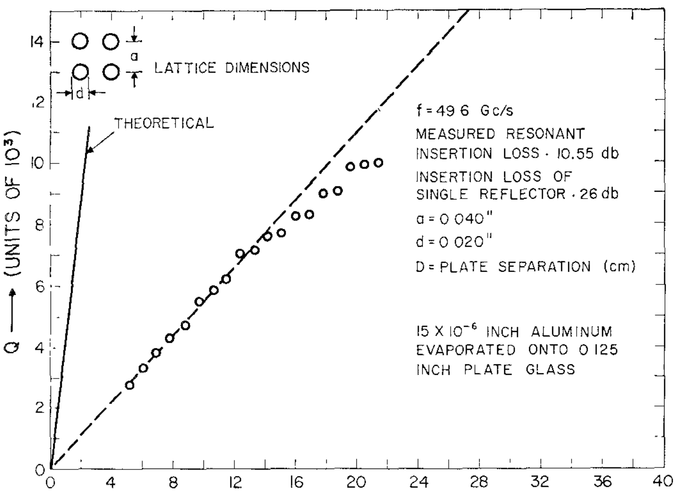

Fig. 4—The measured Q vs mirror separation of a parallel plate Fabry-Perot resonator.

Efficient Power Transfer

For many applications, it is necessary to obtain efficient power transfer into the resonator. With reaction resonators and a single horn serving as both transmitter and receiver, it is first necessary to get good power transfer in order to distinguish the resonance from a large background of reflected power. A transmission resonator is easier to adjust since input and output are naturally separated. If a transmission resonator is used in a maser, weak coupling to the transmitter and close coupling to the detector are desired for optimum sensitivity. Otherwise more of the signal developed by the sample between the plates is wasted in the generator.

In the former case, exactly, and in the latter case, approximately, we may consider one of the two plates to be opaque. To determine the conditions for optimum power transfer to the horn adjacent to the perforated plate, we apply the impedance theory mentioned earlier. This theory assumes the plane wave approximation. At first we shall neglect losses in the perforated plate. This assumption appears to be illogical since a perforated plate might be expected to be intrinsically more lossy than a solid one. However, later considerations will show that, contrary to our intuition, the losses due to this plate do not play an important role. When these losses are neglected, it is more convenient to work with admittance than impedance.

According to basic electromagnetic theory, a solid metallic plane surface at normal incidence can be represented as a normalized admittance of

$$y = g(1 - j) \tag{7}$$

where

$$g = \left(\frac{\sigma}{4\pi\epsilon_0 f}\right)^{1/2} \quad \text{and} \quad \mu = \mu_0$$

and where σ = conductivity of the metal, f is the frequency, and ϵ_0 is the permittivity of empty space. With frequencies in the millimeter wave region and with metals of good conductivity g is a large number, something between 10^3 and 10^4.

As the plane of reference moves away from the metal, the admittance changes in accordance with transmission line theory. It is then possible, in principle, to choose D in such a way that the normalized conductance at the reference plane is unity while the susceptance is positive. Then if the perforated plate is placed here and if the hole pattern is selected in such a way as to make the negative susceptance equal in magnitude to the positive transformed susceptance of the solid plate, a perfect admittance match is obtained.

Because g is so large the Smith Chart cannot be used. For the same reason the analytical expression can be considerably simplified. Applying standard transmission line theory and making various approximations, the following simple results can be obtained. These approximations include 1) neglecting terms of the order of unity and of the order of $\sqrt{g}$ in comparison with terms of the order of g, 2) approximating the tangent of the phase angle by the angle in radians, 3) retaining only lead terms in binominal expansions.

The required value of normalized susceptance for the hole pattern is

$$b = -(2g)^{1/2}. \tag{8}$$

The reflection coefficient of the solid metal surface is

$$\Gamma = \frac{1+j}{g} - 1 \tag{9}$$

and

$$|\Gamma|^2 = 1 - \frac{2}{g} \cdot \tag{10}$$

The required distance D is given by the roots of the equation

$$\tan \beta D = -(1/2g)^{1/2} \tag{11}$$

where

$$\beta = \frac{2\pi}{\lambda_0} \cdot$$

By use of (6) and (7) it can be shown that

$$Q = \frac{\pi m g}{2} \cdot \tag{12}$$

Up to this point losses in the perforated plate have been neglected. There is a theoretical argument which indicates that if the surface admittance of this plate is of the same order of magnitude as that of the solid plate, the effect of these losses is to make only minor changes in the hole diameter and plate separation required for optimum power transfer. Therefore, compensation for these effects can be accomplished by changing slightly these quantities by experiment. Since this theoretical argument is lengthy and probably of little interest, it will not be given here. However, our actual experience indicates that it is justified in practice.

A Parallel Plate Stark Cell

In the design of a Fabry-Perot interferometer for use in observing the Stark effect in millimeter wave molecular spectra we have applied the principles of the previous sections. This device is shown in Figs. 5 and 6. One plate is solid, and the other is composed of a copper foil 0.0015 inch thick with a square array of holes. Both are gold plated. The solid plate is mounted on insulators so that a dc or low-frequency ac voltage can be applied between reflectors.

The ring which supports the insulators is supported by three magnetostriction transducers. These employ nickel armatures 2 inches long and 0.25 inch in diameter. The magnetic circuits are completed with soft iron except for small air gaps. By dissipating a few watts of dc power in the transducer coils it is possible to nearly magnetically saturate the armatures producing a contraction of about 50 microinches. If all three coils are excited, the plate is translated an amount corresponding to detuning the cavity by about the 3-db bandwidth, thus achieving fine tuning. If the trans-

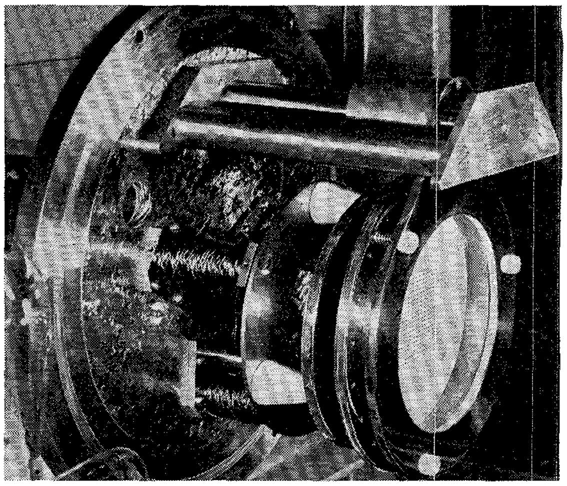

Fig. 5—Parallel plate Stark cell with vacuum cover removed.

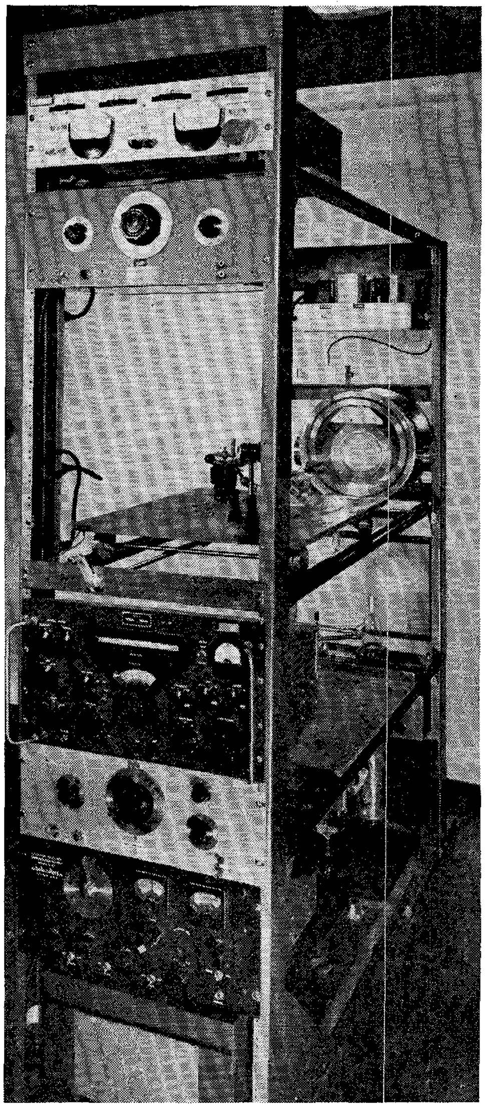

Fig. 6—Assembled view of parallel plate Stark cell showing associated equipment.

ducers are excited differently, this effect may be used to change the alignment. Coarse alignment adjustment is made by three screws holding the perforated reflector to the main frame. Coarse tuning is accomplished by a lead screw in the shaft from which the reflector is supported. The apparatus is provided with a gasketed cover having a plastic front. The lead screw extends through the vacuum envelope allowing coarse tuning adjustment even when the chamber is evacuated.

The foil is stretched on the frame before the insertion of the holes. For the insertion, it is then laid on backing material. The holes used in this interferometer were inserted by a specially made punch, but in other plates with larger holes, drills have been used.

By the use of measurements with calipers, the plates can be made sufficiently parallel to make resonances detectable. The final adjustment makes the resonances as sharp as possible. Putting the vacuum cover in place has only little effect on the response.

Fig. 6 shows the assembly of the interferometer and some of the associated equipment. The assembly employs a frame composed of two standard relay racks tied together by horizontal bars which holds the interferometer, vacuum system, and many of the associated electronic instruments. The horn which feeds the interferometer, as well as the klystron and the associated microwave components, are mounted on a shelf with wheels using two of the horizontal tie bars as tracks. By means of a lead screw adjustable at a panel on each relay rack, the horn-resonator spacing can be conveniently varied to obtain the optimum power transfer when the impedance match is not perfect.

This interferometer was designed with the intent of producing a uniform dc field between the plates. The array of holes occupies a 2-inch square. The horn is approximately 1.5 inches square. Practical experience as well as theory indicate that with close plate spacing the RF field is confined to a cross section slightly larger than the area of the array of holes. The dc field between the plates extends far beyond the RF field, assuring Stark field homogeneity within the active volume. The spacing between the plates can be varied between 7 mm and 45 mm. The widest spacing is slightly less than the size of the hole pattern. For a design center wavelength of 4.29 mm, the m values range from 4 to 10. Consideration of a number of conflicting factors leads to the choice of a moderately small spacing as preferable for the present application. Factors favoring small spacing are 1) relative freedom of inhomogeneity in the Stark field caused by edge effects and 2) relative freedom from pulling of the frequency of spectral lines by the response of the resonator. At close spacings the Q is low and the cavity resonance is broad. The factors favoring large spacing are 1) decrease in relative inhomogeneities in the Stark field caused by the holes and 2) high signal-to-noise ratio because of higher Q. It appears that the latter factors are less important than the former. However, the optimum spacing is to be determined by experiment.

The hole pattern was designed for operation at 3-mm wavelength. A spacing of 0.1 inch was selected as being a convenient value slightly less than the wavelength (0.118 inch). As mentioned earlier a spacing greater than a wavelength is undesirable because of the reinforcement of resonances with maxima off the axis. While the apparatus was being fabricated, it was decided to change the operating wavelength to 4.29 mm (0.169 inch). Under these conditions, for gold, (7) gives $g=2.28\times10^3$, and (8) gives the required susceptance for the holes as -67.4. By (1) the required hole size is 0.029 inch. In order to test the theory the reflection coefficient was determined experimentally and it was found that holes with 0.1 inch spacing but with diameters of 0.037 inch yielded nearly 100 per cent absorption for values of $m=5$ to $m=15$ at a frequency of 70 Gc. Subsequent measurements at 55.2 Gc using the same hole pattern indicate an absorption of approximately 10 per cent. This corresponds to an SWR of 9:1 and gives some indication of the bandwidth characteristics of the array of holes as a coupling device.

Experimentally the degree of impedance mismatch is nearly independent of the horn-to-resonator spacing and of the spacing between reflectors. This latter fact is expected if the preceding theory is valid. If the shunt impedance is defined as the ratio of rms electric field strength to magnetic field strength at a reference plane where the latter is a minimum, it can be inferred that this quantity depends only on $|\Gamma|^2$ and not on m. Therefore it is not an explicit function of Q and increasing m does not change the field strength although it increases Q. In these respects this type of resonator differs from the conventional one.

If the above value of g is substituted into (12), Q/m is calculated to be 3.6×10^3. No attempt was made to make an accurate measurement, but a rough measurement indicated good agreement.

A discussion of the Stark effect for the measurement of voltage and the application of this instrument will be discussed by two of the present authors in another paper.[16]

Spherical Resonators

If a uniform Stark field is not needed and relatively narrow spectral lines are to be investigated, the high Q and small size of the spherical plate resonator recommend its use. The sensitivity of a resonant cavity to small changes in cavity loss is well known[17] to be $\Delta V/V=Q\lambda\alpha/2\pi$ where V is the voltage incident on the detector at cavity resonance and α is the free-space attenuation of the gas sample within the resonator. With a loaded Q of 10^5, it is practical to measure very small

[16] Y. Beers and G. Strine, "The Measurement of Voltage by Use of the Stark Effect, presented at Internat'l Conf. on Precision Measurements, Boulder, Colo.; August 14–17, 1962.

[17] C. H. Townes and A. L. Schawlow, "Microwave Spectroscopy," McGraw-Hill Book Co., Inc., New York, N. Y., Sec. 15-11; 1955.

values of α. The limitation imposed by restricting the resonance bandwidth of the cavity to be several times the natural linewidth of the gas sample is ameliorated by an absence of spectral line broadening due to collisions between the molecules and the cavity walls. The gas pressure can be reduced until pressure broadening is equal to Doppler broadening. For oxygen at room temperature this would occur at about 30 microns pressure and give a linewidth of some 0.25 Mc compared with a cavity bandwidth of 0.6 Mc if the Q were 10^5.

Possible saturation effects in a gas contained in a resonator with such a large Q are not as great as Q alone might imply. As pointed out earlier, in this type of cavity the electric field is not enhanced in the same way as in a simple R-L-C equivalent circuit. From the definition of Q and (6) it can be shown that for a closely coupled resonator with a loaded Q of Q_L the ratio of the electric field within the cavity to that in the transmission line driving it is approximately $(Q_L A_0 / A_r m\pi)^{1/2}$, where A_r and A_0 are the effective areas of the resonator and input transmission line, respectively. This ratio is about unity in the spectrometer to be described.

The spherical plate spectrometer shown in Fig. 7 was designed to isolate the resonator as well as possible from any mechanical forces which might tend to deform it when evacuated. This allows it to be used as a refractometer by measuring the detuning of the cavity as gas is admitted to different pressures. The resonator itself consists of a spherical brass surface of 20-inch radius of curvature facing a flat brass surface nominally 10 inches away. The waveguide feed terminates in the center of the flat plate with an 0.063-inch hole coupling the waveguide to the resonator. The spherical surface is supported on three legs above the flat surface and can be screwed along its axis for tuning. This adjustment can be made through the vacuum container by a retractable finger which can be disengaged to avoid communicating forces to the structure when the pressure is changed. The open aperture of the mirrors is 4.5 inches and the whole resonator fits within a 14-inch length of 6-inch O.D. glass or plastic pipe.

At the top end of the transparent vacuum container a scale is affixed in order to record the axial position of the spherical mirror. The screw thread is metric so that the spectrometer serves also as a precision wavemeter when required. Moving the mirror between two resonances is a translation quite close to a half wavelength which can be read directly in millimeters on the scale.

The brass surfaces were finished to a high polish and precision of a few ten thousandths of an inch. The loaded Q is close to 10^5 at wavelengths of 5 mm. The parameter $a^2/b\lambda$ used by Fox and Li to compute diffraction losses is 1.3 for 5-mm wavelength, suggesting that higher modes than the fundamental TEM_{q00} may be supported. This notation was introduced by Fox and Li.[7] Indeed the axially symmetric TEM_{q01} mode has been identified and behaves qualitatively in all respects as predicted by Fox and Li.

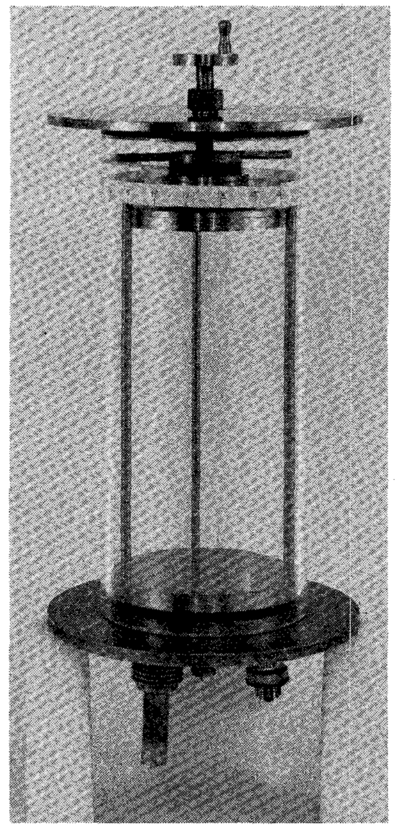

Fig. 7—Spherical plate spectrometer inside transparent vacuum cover.

A spherical plate resonator of small dimensions has been constructed specifically for use as a wavemeter over the waveguide band 50 to 75 Gc. The radius of curvature of the spherical mirrors is only 2 inches, about 10 wavelengths, but it performs quite well. The details of this instrument are discussed elsewhere.[18]

ACKNOWLEDGMENT

The authors want to express thanks to their many colleagues both within and without this laboratory for the many stimulating discussions they have had. In particular we want to express our appreciation to M. Humpal and G. Wichman for the painstaking efforts they took in the fabrication of the resonators discussed here.

[18] R. W. Zimmerer, "New wavemeter for millimeter wavelengths," *Rev. Sci. Instr.*, vol. 33, pp. 858–859; August, 1962.